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[5] أسئلة

For any signal, information is found in frequency component.

Frequency Component (spectrum)



for discrete systems \Rightarrow DFT

$$X(k) = \int_{-\infty}^{\infty} x(n) e^{-j\omega t} dt$$

$$X(k) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$$

Periodic Sequence

$$\omega = \frac{2\pi}{N}$$

$N \Rightarrow$ No. of samples in one cycle.

$0 \rightarrow N-1$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi k n}{N}}$$

1st assignment

program to compute DFT

Input $x(n)$ \Rightarrow sequence, preferred to plot it
Output $x(k)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}}$$

$$= \sum_{n=0}^{N-1} x(n) \underbrace{\left(e^{-j2\pi \frac{1}{N}} \right)^{nk}}_{\omega_n^k}$$

$$k=0 \rightarrow X(0) = x(0)\omega_4^0 + x(1)\omega_4^0 + \dots$$

$$k=1 \rightarrow X(1) = x(0)\omega_4^1 + x(1)\omega_4^1 + x(2)\omega_4^2$$

$$\vdots$$

$$\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(n-1) \end{bmatrix} = \begin{bmatrix} \omega_N^0 & \omega_N^0 & \dots \\ \omega_N^1 & \omega_N^1 & \dots \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(n-1) \end{bmatrix}$$

Ex: 3-Point DFT

$$x(n) = \{1, 2\}$$

Soln

$$X(n) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\omega_3 = e^{-j2\pi/3} = \cos \frac{2\pi}{3} - j \sin \frac{2\pi}{3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\omega_3^{kn} = \begin{matrix} & n=0 & & k=0 \\ & & 1 & 2 \\ \begin{matrix} 1 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega_3^1 & \omega_3^2 \\ 1 & \omega_3^2 & \omega_3^4 \end{bmatrix} \end{matrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 3 \\ 0.5 - j\frac{\sqrt{3}}{2} \\ 0.5 + j\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$|X(K)| = \{3, 1, 1\}$$

$$\angle X(K) = \{0, -60, 60\}$$

Ex: 4 - Points DFT :-

$$X(n) = \{1, -1, 0, 1\}$$

$$N = 4$$

$$X \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$W_4 = e^{-j \frac{2\pi}{4}} = \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} = -j$$

$$W_4^{kn} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = X(K)$$

$$X(K) = \{1, 1+j2, 1, 1-2j\}$$

$$|X(K)| = \{1, \sqrt{5}, 1, \sqrt{5}\}$$

$$\angle X(K) = \{0, 63, 0, -63\}$$

3-Points DFT

$$X(n) = \{-1, 1\}$$

$$X(n) = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$X(K) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$X(K) = \left\{ 0, -\frac{3}{2} - j\frac{\sqrt{3}}{2}, -\frac{3}{2} + j\frac{\sqrt{3}}{2} \right\}$$

$$|X(K)| = \{0, \sqrt{3}, \sqrt{3}\}$$

$$\angle X(K) = \{0, 210, 150\}$$

Continue Assignment :-

* Application depends on DSP

⇒ * General Blocks of application

⇒ * determine which blocks use DS